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Time-dependent *CP* violation in $B^0 \to P^0 P^0 X^0$ decays

Tim Gershon, Masashi Hazumi

High Energy Accelerator Research Organization (KEK), Tsukuba, Japan

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Abstract

We note that in decays of the type $B^0 \to P^0 P^0 X^0$, where P^0 and X^0 represent any *CP* eigenstate spin-0 neutral particles, the final state is a *CP* eigenstate. We consider the possibilities for studying time-dependent *CP* violation in decays of this type at B factories with high luminosity, with particular attention to $B^0 \to K^0_S K^0_S K^0_S$. We also comment on some cases where X^0 has non-zero spin, and decays of the type $B^0 \to P^0 Q^0 X^0$, where the three final state particles are different spin-0 *CP* eigenstates.

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1. Introduction

Recent years have seen a large amount of new experimental information on the phenomena of *CP* violation, one of the least well understood aspects of the Standard Model of particle physics. The observation of mixing-induced *CP* violation in $B^0 \to J/\psi K^0_S$ decays [1,2] and the observation of direct *CP* violation in the neutral kaon system [3] are both in agreement with the predictions of the Kobayashi–Maskawa theory [4]. The KM mechanism is thus established as an effective and elegant description of the weak interactions of the Standard Model quarks.

Nonetheless, it would be premature to claim that *CP* violation is understood. Internationally, a large number of experiments, at various stages from planning to running, are intended to elucidate further information on this most mysterious of phenomena. Testing the validity of the KM mechanism as the sole source of *CP* violation is one of the major experimental challenges in high energy physics today.

Already, there are some tantalizing hints that *CP* violating observables may provide evidence for physics beyond the Standard Model. In particular, the world average for the mixing-induced *CP* violation parameter in $b \to sq\bar{q}$ decays ($q = u, d, s$; e.g., $B^0 \to \phi K^0_S$), is only marginally consistent with that measured in $b \to c\bar{c}s$ decays (e.g., $B^0 \to J/\psi K^0_S$) [5–7], whereas they are predicted to be equal to within a few percent in the Standard Model [8]. Since the former decays are

E-mail addresses: gershon@bmail.kek.jp (T. Gershon), masashi.hazumi@kek.jp (M. Hazumi).

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predominantly governed by flavour changing neutral current penguin amplitudes, they are commonly believed to be sensitive to new physics contributions [9].

The significance of the deviation from the Standard Model prediction will be clarified once more data has been accumulated and the statistical precision of the measurements improved. However, it is essential to try to find additional modes which may be sensitive to the same effect, in order to corroborate the evidence and/or investigate which underlying interaction is responsible. In this spirit, experimental results using the decay modes $B^0 \to \eta' K^0_S$, $B^0 \to K^+K^-K^0_S$ and $B^0 \to K^0_S\pi^0$ have already been produced [10], alongside those for $B^0 \to \phi K^0_S$. Additional modes sensitive to the $b \to s$ penguin amplitude would make extremely welcome additions to this set of measurements.

It is not only the $b \to s$ penguin transition which is of interest. In general any decay mode which is sensitive to *CP* violation, which to date has only been observed in a handful of decay modes, is certainly worth pursuing. Furthermore, any model which proposes new physics effects in $b \to s$ amplitudes via the introduction of a new heavy particle in the loop, may also have an effect on the phase of other “loop” amplitudes, such as the $b \to d$ penguin or B–$\bar{B}$ mixing.

So, we are interested in any B meson decay mode in which *CP* violation, and/or new physics contributions may be investigated. As has been known for some time, neutral B decays to *CP* eigenstates can exhibit *CP* violation with rather straightforward phenomenology [11]. We are also interested in non-*CP* eigenstate final states, where new physics contributions have a relatively clean signal, such as radiative B decays. In this Letter we present some examples of final states which we hope may provide fertile ground for investigating these effects at current and future B factories.

2. *CP* of $B^0 \to P^0P^0X^0$ decays

2.1. Spin-0 X^0

We are considering decays of the form $B^0 \to P^0P^0X^0$. Here B^0 represents a neutral B meson (which can actually be either B^0 or $\bar{B}^0$); P^0 represents any spin-0 neutral particle (we use P^0 since this particle will typically be a pseudo-scalar ($J^P = 0^-$), but scalar particles ($J^P = 0^+$) are also allowed). X^0 also represents any spin-0 neutral particle, which may be either identical to or different to P^0. In what follows, L denotes the angular momentum of the P^0P^0 system, and L' denotes the angular momentum of X^0 relative to the P^0P^0 system.

Due to Bose–Einstein statistics, the P^0P^0 wavefunction must be symmetric, and hence the angular momentum between the two P^0 particles (L) must be even. Therefore, we can write down the *CP* of the P^0P^0 system:

$$CP\big(P^0P^0\big) = C\big(P^0P^0\big) \times P\big(P^0P^0\big)$$
$$= C\big(P^0\big)^2 \times P\big(P^0\big)^2 \times (-1)^L \tag{1}$$
$$= CP\big(P^0\big)^2 = +1, \tag{2}$$

where we have assumed that P^0 is a *CP* eigenstate (i.e., $CP(P^0) = \pm 1$) in the last step.

By conservation of angular momentum in the decay $B^0 \to P^0P^0X^0$, we obtain

$$\mathbf{J}_{B^0} = \mathbf{L} + \mathbf{L}' + \mathbf{S}_{P^0} + \mathbf{S}_{P^0} + \mathbf{S}_{X^0}, \tag{3}$$
$$\mathbf{0} = \mathbf{L} + \mathbf{L}', \tag{4}$$

since the neutral B meson is a spin-0 particle, as are P^0 and X^0. In the above equations, $\mathbf{J}$, $\mathbf{L}$ and $\mathbf{S}$ represent the total, orbital and intrinsic angular momentum, respectively (and elsewhere J, L and S represent their magnitudes). Therefore, the angular momentum between the P^0P^0 system and X^0 (L') must be equal to L, and we can write down the *CP* of the $P^0P^0X^0$ system:

$$CP\big(P^0P^0X^0\big) = CP\big(P^0P^0\big) \times CP\big(X^0\big) \times (-1)^{L'} \tag{5}$$
$$= CP\big(X^0\big). \tag{6}$$

Therefore, provided that both P^0 and X^0 are *CP* eigenstates, the $P^0P^0X^0$ system will also be a *CP* eigenstate, with the same *CP* as the X^0. The possible P^0 are π^0, η, η', f_0, a_0, K^0_S, K^0_L, and D_{CP}; the possible X^0 include all the P^0 candidates, η_c and χ_{c0}. Note that although K^0_S and K^0_L are not actually *CP* eigenstates, the *CP* violation effect in the neutral kaon system is small enough to be ignored. Furthermore, since K^0_S would be reconstructed via its decay to $\pi^+\pi^-$ (or possibly $\pi^0\pi^0$), the final state is actually $(\pi\pi)_K$, which *is* a *CP* eigenstate. Using this rationale we also consider D_{CP} (a neutral D meson reconstructed in a CP eigenstate) as a possible P^0; for completeness, $K_0^{*0}(1430)$ is also a P^0 candidate when reconstructed in a CP eigenstate ($K_0^{*0} \to K_S^0\pi^0$).

We conclude that for any CP eigenstate spin-0 neutral particles P^0 and X^0 which might be reconstructed at a B factory, the final state in $B^0 \to P^0P^0X^0$ decays is a CP eigenstate, and that these modes therefore provide numerous possibilities for the study of time-dependent CP violation.

2.2. X^0 with non-zero intrinsic spin

In the case that X^0 has non-zero angular momentum, the criterion to conserve angular momentum (Eq. (3)) becomes

$$\mathbf{0} = \mathbf{L} + \mathbf{L}' + \mathbf{S}_{X^0}. \tag{7}$$

The P^0P^0 system is required to have even angular momentum, as noted above. In the case that $L = 0$, we find $L' = S_{X^0}$, and the final state has definite CP content. However, we must also include the possibility that $L = 2, 4, \ldots$, where the final state contains both CP even and CP odd components. Although states with large values of L are suppressed due to the angular momentum barrier, some experimental study is required in order to determine the CP composition in these final states. In the case that the spin-0 state of P^0P^0 is dominant, these modes will be particularly worthy of further study.

2.3. $X^0 = \gamma$

We also consider the special case where $X^0 = \gamma$. Since the photon is a massless vector particle, spin $(P^0P^0) = 0$ is forbidden. Therefore, spin-2 is the lowest spin state possible for the P^0P^0 system.

Decays of the form $B^0 \to M^0\gamma$ are of considerable theoretical interest [12]. Here M^0 can be any hadronic self-conjugate CP eigenstate, and $M^0\gamma$ is not required to be a CP eigenstate since this does not affect the time-dependent asymmetry.

Since the P^0P^0 system must contain an even number (0, 2 or 4) of each quark + anti-quark flavour, decays of the form $B^0 \to P^0P^0\gamma$ can only occur via $b \to d\gamma$ transitions. Other decay modes sensitive to the same amplitude (typically $B^0 \to \rho^0\gamma$, $B^0 \to \omega\gamma$) suffer from considerable experimental difficulties. For that reason, modes such as $B^0 \to \eta\eta\gamma$, $B^0 \to \eta'\eta'\gamma$ and $B^0 \to K_S^0K_S^0\gamma$ may be useful to study mixing-induced asymmetries in $b \to d\gamma$ decays.

Since the P^0P^0 system must have spin of at least two, decays of this form may be suppressed due to the angular momentum barrier. However, evidence for a radiative $b \to s$ transition resulting in a tensor meson, has been observed with a branching fraction of $\mathcal{B}(B^0 \to K_2^{*0}(1430)\gamma) = \mathcal{O}(10^{-5})$ [13]. Thus, observation of similar decays mediated by the $b \to d$ transition (e.g., $B \to f_2(1270)\gamma, a_2(1320)\gamma, f_2'(1525)\gamma$) via $P^0P^0\gamma$ final states may soon be within reach of the B factories.

2.4. CP of $B^0 \to P^0Q^0X^0$ decays

It may be noted that the use of Bose–Einstein statistics in the above argument is, in fact, not necessary. If we consider decays of the type $B^0 \to P^0Q^0X^0$ [14], where the three final state particles are different CP eigenstate spin-0 neutral particles, Eq. (3) becomes

$$\mathbf{0} = \mathbf{L} + \mathbf{L}' + \mathbf{S}_{P^0} + \mathbf{S}_{Q^0} + \mathbf{S}_{X^0}, \tag{8}$$

and so we have $L = L'$ as before. (Here $\mathbf{L}$ denotes the angular momentum between P^0 and Q^0, and $\mathbf{L}'$ denotes the angular momentum of X^0 relative to the P^0–Q^0 system.) We can then write down the CP of the final state,

$$\begin{aligned} CP\big(P^0Q^0X^0\big) &= CP\big(P^0\big) \times CP\big(Q^0\big) \times CP\big(X^0\big) \\ &\quad \times (-1)^L \times (-1)^{L'} \qquad (9) \\ &= CP\big(P^0\big) \times CP\big(Q^0\big) \times CP\big(X^0\big). \qquad (10) \end{aligned}$$

This realization greatly increases the number of possible final states. However, in many cases, decays with three different final state particles may have contributions from a number of different amplitudes, complicating the phenomenological interpretation of their time-dependence. Exceptions may arise in the case that two or more of the particles are from the set (π^0, η, η'), or when the final state includes K_S^0 and K_L^0; nevertheless, we restrict ourselves to the case $P^0 = Q^0$ in the main part of this Letter. Note, however, one useful consequence of this observation: in the case of studying time-dependent CP violation in a decay such as $B^0 \to \eta' K^{*0}$ with $K^{*0} \to K_S^0\pi^0$, any background from $B^0 \to \eta' K_S^0\pi^0$, either non-resonant

Table 1
Possible $B^0 \to P^0 P^0 X^0$ final states. Underlined modes are discussed in detail in the text. The doubly underlined mode $B^0 \to K_S^0 K_S^0 K_S^0$ has already been observed

	P^0							
X^0	π^0	η	η'	f_0	a_0	K_S^0	K_L^0	D_{CP}
π^0	$\underline{\pi^0\pi^0\pi^0}$	$\eta\eta\pi^0$	$\eta'\eta'\pi^0$	$f_0 f_0\pi^0$	$a_0 a_0\pi^0$	$\underline{K_S^0 K_S^0\pi^0}$	$K_L^0 K_L^0\pi^0$	$D_{CP}D_{CP}\pi^0$
η	$\pi^0\pi^0\eta$	$\eta\eta\eta$	$\eta'\eta'\eta$	$f_0 f_0\eta$	$a_0 a_0\eta$	$K_S^0 K_S^0\eta$	$K_L^0 K_L^0\eta$	$D_{CP}D_{CP}\eta$
η'	$\pi^0\pi^0\eta'$	$\eta\eta\eta'$	$\eta'\eta'\eta'$	$f_0 f_0\eta'$	$a_0 a_0\eta'$	$K_S^0 K_S^0\eta'$	$K_L^0 K_L^0\eta'$	$D_{CP}D_{CP}\eta'$
f_0	$\pi^0\pi^0 f_0$	$\eta\eta f_0$	$\eta'\eta' f_0$	$f_0 f_0 f_0$	$a_0 a_0 f_0$	$K_S^0 K_S^0 f_0$	$K_L^0 K_L^0 f_0$	$D_{CP}D_{CP} f_0$
a_0	$\pi^0\pi^0 a_0$	$\eta\eta a_0$	$\eta'\eta' a_0$	$f_0 f_0 a_0$	$a_0 a_0 a_0$	$K_S^0 K_S^0 a_0$	$K_L^0 K_L^0 a_0$	$D_{CP}D_{CP} a_0$
K_S^0	$\underline{\pi^0\pi^0 K_S^0}$	$\underline{\eta\eta K_S^0}$	$\underline{\eta'\eta' K_S^0}$	$f_0 f_0 K_S^0$	$a_0 a_0 K_S^0$	$\underline{\underline{K_S^0 K_S^0 K_S^0}}$	$K_L^0 K_L^0 K_S^0$	$D_{CP}D_{CP} K_S^0$
K_L^0	$\pi^0\pi^0 K_L^0$	$\eta\eta K_L^0$	$\eta'\eta' K_L^0$	$f_0 f_0 K_L^0$	$a_0 a_0 K_L^0$	$\underline{K_S^0 K_S^0 K_L^0}$	$K_L^0 K_L^0 K_L^0$	$D_{CP}D_{CP} K_L^0$
D_{CP}	$\pi^0\pi^0 D_{CP}$	$\eta\eta D_{CP}$	$\eta'\eta' D_{CP}$	$f_0 f_0 D_{CP}$	$a_0 a_0 D_{CP}$	$\underline{K_S^0 K_S^0 D_{CP}}$	$K_L^0 K_L^0 D_{CP}$	
η_c	$\pi^0\pi^0\eta_c$	$\eta\eta\eta_c$	$\eta'\eta'\eta_c$	$f_0 f_0\eta_c$	$a_0 a_0\eta_c$	$K_S^0 K_S^0\eta_c$	$K_L^0 K_L^0\eta_c$	
χ_{c0}	$\pi^0\pi^0\chi_{c0}$	$\eta\eta\chi_{c0}$	$\eta'\eta'\chi_{c0}$	$f_0 f_0\chi_{c0}$	$a_0 a_0\chi_{c0}$	$K_S^0 K_S^0\chi_{c0}$	$K_L^0 K_L^0\chi_{c0}$	

or produced via some other intermediate state, can be treated as signal since it has the same *CP* eigenstate. (Possible contributions from different amplitudes may, however, pollute the interpretation of the result.)

3. Consideration of specific $B^0 \to P^0 P^0 X^0$

In the remainder of this Letter, we consider specific decays of the form $B^0 \to P^0 P^0 X^0$ where both P^0 and X^0 are spin-0 neutral particles. As is clear from Table 1, there are an enormous number of modes of this type. We should consider which of these are most likely to be useful, in terms of both a clean phenomenological interpretation, and of the possibility of obtaining a sufficiently large sample to study *CP* violating effects. This last point is problematic, since most of these decay modes have not yet been observed, and we know of no reliable technique with which to estimate the three body branching fractions. Where necessary, we guess the approximate size of the branching fractions based on existing experimental measurements. As a corollary, we note that measurements of the branching fractions of these modes will provide useful information on the nature of hadronic *B* decays.

3.1. $X^0 = \eta_c, \chi_{c0}$

In the study of $B \to$ charmonium decays, $c\bar{c} = J/\psi$ is usually preferred over $c\bar{c} = \eta_c$ or χ_{c0}, owing to the former's clean signal and high reconstruction efficiency in the decay $J/\psi \to l^+l^-$, $l = e, \mu$. Although in principle $B^0 \to \eta_c K_S^0 K_S^0$ or $B^0 \to \eta_c \pi^0\pi^0$, and equivalent modes for χ_{c0}, could be used to investigate the $b \to c\bar{c}d$ transition, as shown in Fig. 1, in practice the backgrounds will be rather large and the reconstruction efficiency rather low. We do not consider these modes further.

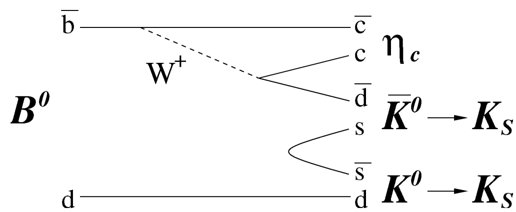

Fig. 1. Feynman diagram for $B^0 \to \eta_c K_S^0 K_S^0$, which is the same as that for $B^0 \to \eta_c \pi^0$ with additional $s\bar{s}$ production. Only the (dominant) tree contribution is shown.

3.2. $P^0 = D_{CP}$ or $X^0 = D_{CP}$

Decays of the form $B \to DDK$ have previously been shown to be useful in probing the $b \to c\bar{c}s$ amplitude and in resolving the ambiguity in ϕ_1 from measurements of $\sin 2\phi_1$ [15]. There is no immediately apparent benefit from requiring the *D* meson to be reconstructed in a *CP* eigenstate. The same applies for $B \to DD\pi$, which probes $b \to c\bar{c}d$.

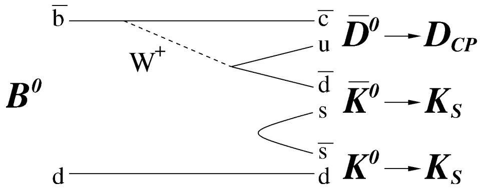

Fig. 2. Feynman diagrams for $B^0 \to D_{CP} K_S^0 K_S^0$, which are the same as those for $B^0 \to D_{CP}\pi^0$ with additional $s\bar{s}$ production.

The decay mode $B^0 \to D_{CP}\pi^0$ has been proposed to determine the B–$\bar{B}$ mixing phase with an order of magnitude less theoretical uncertainty than exists in $B^0 \to J/\psi K_S^0$ [16]. Due to this theoretical cleanliness, this mode is also sensitive to some new physics models [9]. The decay $B^0 \to D_{CP} K_S^0 K_S^0$ is mediated by the same diagrams, with additional $s\bar{s}$ production, as shown in Fig. 2.

At B factories, the displaced vertex and the narrow width of the K_S meson provide a rather clean experimental signature. This leads to lower backgrounds from random combinations of particles than in corresponding modes containing π^0s. Therefore, although $B^0 \to D_{CP} K_S^0 K_S^0$ is expected to have a smaller branching fraction than $B^0 \to D_{CP}\pi^0$, the sensitivity to time-dependent observables may be comparable.

Similar decay modes $B \to DKK$ have previously been observed, with branching fractions of the order of 10^{-4} [17]. The production may be dominated by intermediate spin-1 KK resonances, but a significant non-resonant contribution is not ruled out. In this case, it may be possible to observe the decay $B^0 \to \bar{D}^0 K_S^0 K_S^0$, with the current B factory statistics, using the decay $\bar{D}^0 \to K^+\pi^-$. As with $B^0 \to D_{CP}\pi^0$, the requirement to reconstruct the D meson in a CP eigenstate leads to a further reduction in statistics; however, it may be possible to perform time-dependent measurements of $B^0 \to D_{CP} K_S^0 K_S^0$ in the future.

3.3. P^0 and $X^0 = K_S^0, K_L^0$

As previously mentioned, the possibility of a new physics signal in the time-dependence of $B^0 \to \phi K_S^0$ decays [5] is one of the most exciting results in high energy physics today. To fully investigate this possibility, as many $b \to s\bar{s}s$ modes as possible need to be utilized. In addition to ϕK_S^0, $\eta' K_S^0$ and $K^+K^-K_S^0$ have been used to date. In the latter case, an isospin analysis indicates that after removing events where the K^+K^- combination may have originated from a ϕ meson, the remaining $K^+K^-K_S^0$ candidates are predominantly $CP = +1$, which in turn indicates that the K^+K^- system has even angular momentum [18]. Since any even angular momentum state which decays to K^+K^- should also decay to $K_S^0 K_S^0$ and $K_L^0 K_L^0$, this further suggests that $B^0 \to K_S^0 K_S^0 K_S^0$ and $B^0 \to K_L^0 (K_L^0 K_S^0)_{\text{non-}\phi}$ may have reasonable branching fractions. Indeed, $B^0 \to K_S^0 K_S^0 K_S^0$ has been observed, as shown in Fig. 3.

In terms of phenomenology, these decays have an advantage over $B^0 \to K^+K^-K_S^0$, since the latter suffers a contribution from the $b \to u$ tree diagram, which has a different weak phase. Here, however, there is no u quark in the final state. The $b \to su\bar{u}$ tree diagram followed by rescattering into $sd\bar{d}$ or $ss\bar{s}$ is OZI-suppressed. Therefore these are almost pure penguin decays. There can be contributions from both $b \to ss\bar{s}$ with additional $d\bar{d}$ production, and $b \to sd\bar{d}$ with additional $s\bar{s}$ production, but these diagrams have the same weak phase. Any new physics contribution expected in $B^0 \to \phi K_S^0$ also affects $B^0 \to K_S^0 K_S^0 K_S^0$ and $B^0 \to K_L^0 (K_L^0 K_S^0)_{\text{non-}\phi}$ (and, for completeness, $B^0 \to \phi K_L^0$) and in the absence of new physics all should exhibit the same CP violating effects as $J/\psi K_S^0$.

Turning to experimental considerations, we note that final states containing K_L^0 tend to suffer large backgrounds. This appears to be an insurmountable problem for B decays containing two or more K_L^0 mesons in the final state at any planned experiment, and so we do not consider $B^0 \to K_S^0 K_L^0 K_L^0$ or $B^0 \to K_L^0 K_L^0 K_L^0$ further.

The first results on time-dependent analyses from the B factories have used B decays with final states including particles which may be considered to originate from the B decay vertex, e.g., $J/\psi K_S^0$, $\pi^+\pi^-$, ϕK_S^0. For these modes, the performances of the vertexing subsystems and algorithms are well understood [1,2,19]. Results using modes where the primary

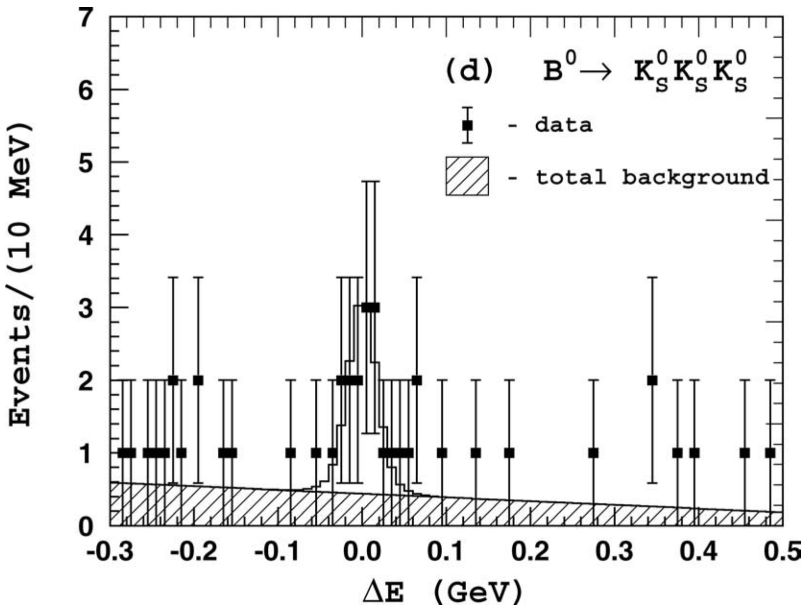

Fig. 3. Observation of $B^0 \to K^0_S K^0_S K^0_S$, from [18]. The plotted variable, ΔE is the difference between the reconstructed and expected B candidate energies. From 78 fb^{-1} of data recorded on the $\Upsilon(4S)$ resonance, a signal yield of $12.2^{+4.5}_{-3.8}$ is obtained, leading to a branching fraction of $\mathcal{B}(B^0 \to K^0_S K^0_S K^0_S) = (4.2^{+1.6}_{-1.3} \pm 0.8) \times 10^{-6}$.

B daughters have a lifetime which is not negligible, e.g., $D^{*+}D^-$, $D^{*+}D^{*-}$, have also appeared [20]. In these analyses, the displacement of the daughter tracks must be taken into account when measuring the vertex position. Recently, the BaBar Collaboration have succeeded to measure the B decay vertex in $B^0 \to K^0_S \pi^0$ [10]. The possibility to obtain vertex information from K^0_S mesons makes time-dependent analyses of $B^0 \to K^0_S K^0_S K^0_S$ and $B^0 \to K^0_S K^0_S K^0_L$ feasible.

3.3.1. $B^0 \to K^0_S K^0_S K^0_S$

As pointed out above, this mode has been observed [18]. From 78 fb^{-1} of data recorded on the $\Upsilon(4S)$ resonance, $12.2^{+4.5}_{-3.8}$ signal events are found. From Fig. 3, we can count the number of candidates in the region $-0.1\ \text{GeV} < \Delta E < 0.1\ \text{GeV}$, to estimate the signal-to-background ratio (S/B). There are 21 candidates in this region giving $S/B \gtrsim 1$.

The efficiency to obtain a K^0_S vertex reflects the probability for the K^0_S to decay inside the vertex detector, and so depends on the K^0_S momentum and on the size of the vertex detector. In a three body $B^0 \to K^0_S K^0_S K^0_S$ decay, at least one K^0_S must have fairly low momentum in the B^0 rest frame. Therefore, we expect a high vertex efficiency for $B^0 \to K^0_S K^0_S K^0_S$, if the vertex efficiency for $B^0 \to K^0_S \pi^0$ (where the K^0_S has high momentum) is moderate. In the time-dependent analysis of $B^0 \to K^0_S \pi^0$ by the BaBar Collaboration, a vertex efficiency of 65% is obtained [10]. Therefore, we expect that vertex efficiencies for $B^0 \to K^0_S K^0_S K^0_S$ of close to 100% may be obtained at B factories with similar vertex detector geometry.

Noting that the purity of the $K^0_S K^0_S K^0_S$ signal is comparable to that of $\eta' K^0_S$, the precision on the parameter $S_{K^0_S K^0_S K^0_S}$ can be predicted [21]. From 140 fb^{-1} Belle has 421 $B \to \eta' K^0_S$ candidates, and measures the error on $S_{\eta' K^0_S}$ to be $\delta S_{\eta' K^0_S} = 0.27$, hence the statistical sensitivity from $B^0 \to K^0_S K^0_S K^0_S$ with 5 ab^{-1} of data is estimated as $\delta S_{K^0_S K^0_S K^0_S} \sim 0.14$.

In fact, an initial time-dependent study of this mode may be practicable at present B factories as a number of improvements are possible:

(1) By optimizing event selection criteria in each flavor tagging category. Events with good flavor tag-

ging quality, which contribute most to the statistical significance of the result, tend to have smaller backgrounds from continuum processes ($e^+e^- \to q\bar{q}$, $q = u, d, s, c$).

(2) By optimizing the K_S^0 selection for this analysis, some improvement in the K_S^0 efficiency may be obtained. The $K_S^0 K_S^0 K_S^0$ efficiency will benefit by the improvement to the third power.

(3) By including events where one K_S^0 is reconstructed in the $\pi^0\pi^0$ final state. Since $\mathcal{B}(K_S^0 \to \pi^+\pi^-) \approx 2\mathcal{B}(K_S^0 \to \pi^0\pi^0)$, we expect

$$N_{\mathrm{true}}\big(B^0 \to 2(\pi^+\pi^-)_{K_S^0}(\pi^0\pi^0)_{K_S^0}\big) \approx \frac{3}{2} N_{\mathrm{true}}\big(B^0 \to 3(\pi^+\pi^-)_{K_S^0}\big), \tag{11}$$

where N_{true} indicates the number of B mesons decaying in the specified manner, i.e., no reconstruction efficiency is taken into account.

To estimate the ratio of reconstruction efficiencies, $\epsilon_{K_S^0 \to \pi^+\pi^-}/\epsilon_{K_S^0 \to \pi^0\pi^0}$, we refer to the most recent measurement of $\sin 2\phi_1$ [22] by the Belle Collaboration, in which 1997 $B^0 \to J/\psi(\pi^+\pi^-)_{K_S^0}$ candidates and 288 $B^0 \to J/\psi(\pi^0\pi^0)_{K_S^0}$ candidates are reconstructed. Taking background fractions into account

$$\frac{\epsilon_{K_S^0 \to \pi^+\pi^-}}{\epsilon_{K_S^0 \to \pi^0\pi^0}} \approx \frac{1}{2} \frac{1997 \times 0.976}{288 \times 0.82} \approx 4.1, \tag{12}$$

and so we expect to find the reconstructed numbers of events (N_{rec}) to be approximately related by

$$N_{\mathrm{rec}}\big(B^0 \to 2(\pi^+\pi^-)_{K_S^0}(\pi^0\pi^0)_{K_S^0}\big) \approx 0.36\, N_{\mathrm{rec}}\big(B^0 \to 3(\pi^+\pi^-)_{K_S^0}\big). \tag{13}$$

Although when one K_S^0 is reconstructed via $\pi^0\pi^0$, both S/B and ϵ_{vtx} will be degraded, including these events will nonetheless help to reduce the statistical error.

Although most of the background for this decay is expected to originate from continuum processes, care will have to be taken to reject background from $b \to c$ transitions. Whilst $\eta_c \to K_S^0 K_S^0$ and $J/\psi \to K_S^0 K_S^0$ are forbidden due to parity and CP conservation, respectively, some other charmonium states (e.g., $B^0 \to (K_S^0 K_S^0)_{\chi_{c0}} K_S^0$), may cause a background. Such contributions can be measured from amplitude (Dalitz plot) analyses of $B^0 \to K_S^0 K_S^0 K_S^0$ and other $B \to KKK$ decays, or estimated from known branching fractions. For example, if we assume that $\mathcal{B}(B^0 \to \chi_{c0} K_S^0) = (1/2)\mathcal{B}(B^+ \to \chi_{c0} K^+) = (1/2)(6.0^{+2.1}_{-1.8} \pm 1.1) \times 10^{-4}$ [23], where the factor of 1/2 is due to $\mathcal{B}(K^0 \to K_S^0)$, and take $\mathcal{B}(\chi_{c0} \to K_S^0 K_S^0) = (2.0 \pm 0.6) \times 10^{-3}$ [24], we estimate the background level, $B^0 \to (K_S^0 K_S^0)_{\chi_{c0}} K_S^0 \sim 0.1 \times \mathcal{B}(B^0 \to K_S^0 K_S^0 K_S^0)$. Backgrounds of this type can be rejected using invariant mass requirements.

To summarize this subsection, $B^0 \to K_S^0 K_S^0 K_S^0$ is (a) theoretically clean, (b) very interesting, (c) already observed, (d) an analysis which requires a great deal of further work!

3.3.2. $B^0 \to K_S^0 K_S^0 K_L^0$

We can compare $B^0 \to K_S^0 K_S^0 K_L^0$ to $B^0 \to K_S^0 K_S^0 K_S^0$ again by using information from the most recent measurement of $\sin 2\phi_1$ [22] by the Belle Collaboration, and comparing the numbers of candidates of $J/\psi(\pi^+\pi^-)_{K_S^0}$ and $J/\psi K_L^0$, taking background fractions into account:

$$\epsilon_{K_S^0 \to \pi^+\pi^-}/\epsilon_{K_L^0} \approx \frac{1997 \times 0.976}{2332 \times 0.60} \approx 1.4. \tag{14}$$

Therefore, assuming equal branching fractions, we might expect almost as many $B^0 \to K_S^0 K_S^0 K_L^0$ as $B^0 \to K_S^0 K_S^0 K_S^0$ events, albeit with worse S/B and ϵ_{vtx}. As above, there may be some gain by including events where one K_S^0 is reconstructed via $\pi^0\pi^0$. To know the feasibility of time-dependent CP analyses using $K_S^0 K_S^0 K_L^0$ requires more detailed knowledge of the size of the background.

Since this final state has the opposite CP to $K_S^0 K_S^0 K_S^0$, there is a significant benefit (not least in the presentation of results) in studying these two modes in conjunction with each other. However, there is an additional constraint which may be used in this case. Recalling that the measurement of the CP composition of $B^0 \to (K^+K^-)_{\text{non-}\phi} K_S^0$ as predominantly CP even suggests a reasonably large branching fraction for $B^0 \to (K_S^0 K_S^0)_{\text{non-}\phi} K_S^0$; by the same argument a rather small branching fraction for $B^0 \to (K_S^0 K_L^0)_{\text{non-}\phi} K_S^0$ is expected. Since $\phi \to K_S^0 K_L^0$ is known to have a large branching fraction, we therefore have nothing to lose by imposing a ϕ mass constraint. Since the ϕ resonance is rather narrow, this constraint may be extremely useful in rejecting background. Further experimental investigation is required.

3.4. $P^0 = \pi^0, \eta, \eta'$ and $X^0 = K_S^0, K_L^0$

The decay $B^+ \to \phi\phi K^+$ has previously been pointed out to be sensitive to possible new physics contributions [25], and has recently been observed [26]. Similarly, $B^0 \to P^0 P^0 K_S^0$ modes where P^0 has an $s\bar{s}$ component, may also be sensitive to new physics. As the branching fractions for the decays $B \to \eta' K$ and $B \to \eta' X_S$ are known to be larger than theoretical expectations, $B \to \eta'\eta' K$ decays appear particularly interesting. The dominant amplitude for these decays is the $b \to s$ penguin, although other contributions are possible. Thus, studies of the time-dependence of $B^0 \to \eta'\eta' K_S^0$ can also help to probe for new physics phases in these amplitudes.

As mentioned above, there has recently been some activity on time-dependent analysis of $B^0 \to K_S^0\pi^0$ [10], which has contributions from the $b \to s\bar{d}d$ penguin and $b \to s\bar{u}u$ tree and penguin amplitudes. By adding a $u\bar{u}$ or $d\bar{d}$ pair as appropriate, the same processes can result in the $K_S^0\pi^0\pi^0$ final state. In the three body decay mode the vertexing efficiency should be improved compared to $K_S^0\pi^0$ since the K_S^0 will have lower momentum. (Assuming a phase space distribution of the $K_S^0\pi^0\pi^0$ decay products, we expect the vertexing inefficiency to be reduced by 10–20% compared to $K_S^0\pi^0$.)

Measured branching fractions for $B^+ \to K^+\pi^+\pi^-$ and $B^0 \to K_S^0\pi^+\pi^-$ [18] suggest that $B^0 \to K_S^0\pi^0\pi^0$ may have a larger branching fraction than $B^0 \to K_S^0\pi^0$. Furthermore, an amplitude analysis of $B^+ \to K^+\pi^+\pi^-$ has shown that there are large contributions from $B^+ \to K_0^*(1430)^0\pi^+$, $B^+ \to f_0(980)K^+$ and a non-resonant source [27]. In each case, performing an isospin rotation on the spectator quark leads to an amplitude which contributes to the $B^0 \to K_S^0\pi^0\pi^0$ final state. Therefore, we expect that the branching fraction for $B^0 \to K^0\pi^0\pi^0$ should be comparable to those for $B^0 \to K^0\pi^+\pi^-$ and $B^+ \to K^+\pi^+\pi^-$ (which are $\sim 5 \times 10^{-5}$).

Large backgrounds would be expected in this mode, but these appear to be experimentally controllable. Continuum background can be handled in the usual manner, contributions which proceed via charmed intermediate states (e.g., $B^0 \to \bar{D}^0\pi^0$ or $B^+ \to \bar{D}^0\rho^+$ with $\bar{D}^0 \to K^0\pi^0$) can be removed using two-particle invariant mass requirements, and other charmless B decays (e.g., $B^0 \to K_S^0\pi^0$) are intrinsically small since their branching fractions are less than that expected for $B^0 \to K^0\pi^0\pi^0$. Background from $B^0 \to K^{*0}\gamma$ with $K^{*0} \to K^0\pi^0$ is also expected to be small, and can be further reduced making a requirement on the energy asymmetry of the clusters of the neutral pion candidates.

Thus $B^0 \to K_S^0\pi^0\pi^0$ may provide another handle to probe the $b \to s$ transition. Using the branching fraction given above, and taking subdecays and efficiencies into account, we expect the statistical errors of the time-dependent parameters in this mode will be comparable to those obtained from $B^0 \to K_S^0\pi^0$. Thus we anticipate a precision of $\delta S_{K_S^0\pi^0\pi^0} \sim \delta S_{K_S^0\pi^0} \sim 0.10$ will be obtained with 5 ab^{-1} of data [21].

3.5. $P^0 = K_S^0, K_L^0$ and $X^0 = \pi^0, \eta, \eta'$

Charmless B decays to final states containing an even number of $s\bar{s}$ quarks are suppressed in the Standard Model. These final states can be produced either via a $b \to d$ penguin transition, or by $s\bar{s}$ production following a decay to a final state containing no strange quarks, and both of these amplitudes are rather small. No such decay has been observed to date (although evidence for $B^+ \to K^+K^-\pi^+$ has been found [18]). Nonetheless, branching fraction estimates for some hadronic $b \to d$ transitions suggest these will soon be within reach of the B factories, creating some interesting possibilities. The decay $B^0 \to K_S^0K_S^0\pi^0$ may be mediated by the $b \to d\bar{s}s$ penguin (e.g., $B^0 \to K_S^0K^{*0}$ with $K^{*0} \to K_S^0\pi^0$), or by the same diagrams which mediate $B^0 \to \pi^0\pi^0$ followed by $s\bar{s}$ production. In either case, interesting observables may be investigated, but some effort will be required to disentangle the contributing amplitudes.

3.6. P^0 and $X^0 = \pi^0, \eta, \eta'$

Once again noting that final states containing even numbers of $s + \bar{s}$ quarks are suppressed, any combination of $P^0, X^0 = (\pi^0, \eta, \eta')$ should be produced via $u\bar{u}$ and $d\bar{d}$ components only. Such two-body final states are used to determine ϕ_2, one of three angles of the unitarity triangle, and the three body final states also contain useful information, in principle. One interesting scenario would be modes which are produced by the same amplitudes as $B^0 \to \pi^0\pi^0$, but for which it is possible to reconstruct a vertex position, and hence study the time-dependence. Additional $s\bar{s}$ production can result in $K^0_S K^0_S \pi^0$; additional $d\bar{d}$ or $u\bar{u}$ production may result in $\eta^{(\prime)}\eta^{(\prime)}\pi^0$. The first evidence for $B^0 \to \pi^0\pi^0$ has recently been seen [28], with a branching fraction of around $\mathcal{B}(B \to \pi^0\pi^0) \approx 2 \times 10^{-6}$. This suggests that extremely large data samples would be necessary in order to perform a time-dependent analysis on these final states.

The angle ϕ_2 can also be determined from the Dalitz plot of $B^0 \to \pi^+\pi^-\pi^0$, or from an isospin analysis of $B \to \rho\pi$ decays [29]. It has been suggested that these analyses may be complicated by the presence of a broad scalar σ resonance decaying to $\pi^+\pi^-$ [30]. The study of $B^0 \to \pi^0\pi^0\pi^0$ could limit the size of this effect, since any such resonance must decay also to $\pi^0\pi^0$. Therefore, it could limit the contribution from this class of background to the $B \to \rho\pi$ final states ($\rho^+\pi^-$, $\rho^-\pi^+$, $\rho^+\pi^0$, $\rho^0\pi^+$, $\rho^0\pi^0$), all of which have now been observed [31,32], except for $B^0 \to \rho^0\pi^0$ for which evidence has been uncovered [32].

4. Summary

It is clear that there are an enormous number of possible $B^0 \to P^0P^0X^0$ decay modes which could yield important information given enough integrated luminosity. Time-dependent studies of many of these modes appear experimentally feasible with the large data samples which can be obtained at a “super B factory”. For modes which use multiple photons in the reconstruction, the environment at e^+e^- machines is preferable to that at hadron colliders. Furthermore, it is only at e^+e^- machines that precise knowledge of the interaction point can be obtained to allow the B vertex to be reconstructed from a single K^0_S meson. Reconstruction of the B vertex from multiple K^0_S mesons (e.g., in $B^0 \to K^0_S K^0_S K^0_S$) at hadron colliders, appears experimentally challenging, but not impossible.

Of the possible $B^0 \to P^0P^0X^0$ decays, $B^0 \to K^0_S K^0_S K^0_S$ appears to be an extremely promising mode to study the $b \to s\bar{s}s$ transition and search for phases from new physics. First results with this mode may be obtained at the present B factories in the near future. In addition, $B^0 \to D_{CP} K^0_S K^0_S$, $B^0 \to \eta'\eta' K^0_S$, $B^0 \to \pi^0\pi^0 K^0_S$ and $B^0 \to \pi^0\pi^0\pi^0$ appear deserving of further study.

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